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Multiple diffraction of plane waves by a soft/hard strip

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Abstract. A uniform asymptotic high-frequency solution is developed for the problem of diffraction of plane waves by a strip which is soft at one side and hard on the other. The related three-part boundary value problem is formulated into a "modified matrix Wiener-Hopf equation". By using the known factorization of the kernel matrix through the Daniele-Khrapkov method, the modified matrix Wiener-Hopf equation is first reduced to a pair of coupled Fredholm integral equations of the second kind and then solved by iterations. An interesting feature of the present solution is that the classical Wiener-Hopf arguments yield unknown constants which can be determined by means of the edge conditions.

1. Introduction

The present work deals with the multiple diffraction of plane acoustic waves by a strip whose upper face is simulated by a sound-soft boundary condition while the lower face is supposed to be sound-hard.

As is well known, diffraction problems involving a strip geometry constitute a three-part mixed boundary value problem which rigorously may be formulated via a triple integral equation approach that yields in general a modified Wiener-Hopf equation (MWHE) of the form

$$G(\alpha)P(\alpha) + \Phi^-(\alpha) + e^{i\alpha l}\Phi^+(\alpha) = f(\alpha) \tag{1}$$

Here, l is the width of the strip defined by $S = \{x \in (0,l)\,,\ y = 0\,,\ z \in (-\infty,\infty)\}$ while $G(\alpha)$ and $f(\alpha)$ are known functions, regular in a certain strip $\tau_+ < \mathrm{Im}\,\alpha < \tau_-$.The unknown functions $\Phi^+(\alpha)$ and $\Phi^-(\alpha)$ are regular in the half-planes $\mathrm{Im}\,\alpha > \tau_+$ and $\mathrm{Im}\,\alpha < \tau_-$, respectively, whereas $P(\alpha)$ is an entire function.

The MWHE appearing in (1) is either a scalar or a matrix one according to the boundary conditions to be satisfied on both faces of the strip being symmetrical or not. As it was shown by Jones [1,Sec.9.12], the solution of the MWHE in (1) can always be reduced to the solution of two uncoupled Fredholm integral equations of the second kind with kernel $1/(\alpha+\beta)$. To find an approximate solution to these equations Jones introduced an iterative approach [1,Sec.9.12] which was later applied to the analysis of different kinds of material strips resulting in a single or two simultaneous MWHEs which can be decoupled elementarily [2,3,4].

The solution of the MWHE in (1) requires one to express the kernel function $G(\alpha)$ as the product of two functions, say $G^+(\alpha)$ and $G^-(\alpha)$, which are regular, free of zeros and of algebraic growth at infinity in the half-planes $\mathrm{Im}\,\alpha > \tau_+$ and $\mathrm{Im}\,\alpha < \tau_-$, respectively (Wiener-Hopf factorization). If the functions involved in (1) are scalar, the Wiener-Hopf factorization can easily be achieved by taking the logarithm of the function $G(\alpha)$ and, with the aid of Cauchy's Theorem, by writing it as the sum of two functions analytic in $\mathrm{Im}\,\alpha > \tau_+$ and $\mathrm{Im}\,\alpha < \tau_-$, respectively. The appropriate factors can then be obtained by taking their

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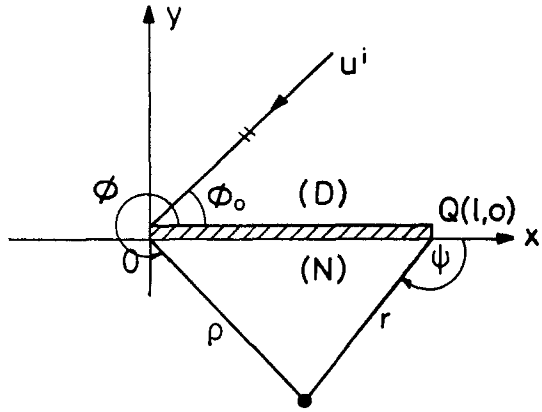

Fig. 1. Geometry of the diffraction problem.

exponentials. However, this procedure makes use of the commutative properties of the factors, and of course cannot be generalized to the matrix case which involves no commutative algebras. This is the main source of difficulty in solving matrix MWHEs.

Although the Wiener-Hopf factorization of an arbitrary matrix still remains at present an open problem, a significant amount of progress has been achieved in the last years for a restricted class of matrices.

The Wiener-Hopf-Hilbert method introduced by Hurd [5] is powerful when the kernel matrix $G(\alpha)$ has only branch-point singularities. It can be seen that this approach includes also the case stated by Rawlins and Williams [6]. A more general procedure which is applicable to the cases for which the kernel matrix has only poles or poles and branch-point singularities is given by Khrapkov [7], Daniele [8], Rawlins[9] and Jones[10].

The matrix MWHE related to the plane wave diffraction by a soft/hard strip which we consider in this work, belongs to the class for which both the Wiener-Hopf-Hilbert and the Daniele-Khrapkov methods are applicable. In the present paper the Wiener-Hopf factorization of the kernel matrix is performed via the Daniele-Khrapkov method and the matrix MWHE is reduced to a pair of coupled Fredholm integral equations of the second kind. At this stage, unknown constants resulting from the application of Liouville's theorem are introduced. Approximate solutions to the coupled integral equations are then obtained by iterations and the constants appearing in the solution are determined by taking into account the behaviour of the scattered field near the edges. Uniform asymptotic expressions for the diffracted fields are derived up to and including second-order interaction terms. Some numerical results concerning the variation of the singly and doubly diffracted fields versus the observation angle are presented for different values of the strip width. Furthermore the doubly diffracted fields derived in this work are compared with those of completely rigid and completely soft strips.

A time factor $e^{-i\omega t}$ is assumed and suppressed throughout the paper.

2. Formulation of the Mixed-Boundary-Value Problem

The geometry of the diffraction problem is depicted in Fig. 1. A plane acoustic wave

$$u^i(x,y) = \exp\left[-ik(x\cos\phi_0 + y\sin\phi_0)\right] \tag{2}$$

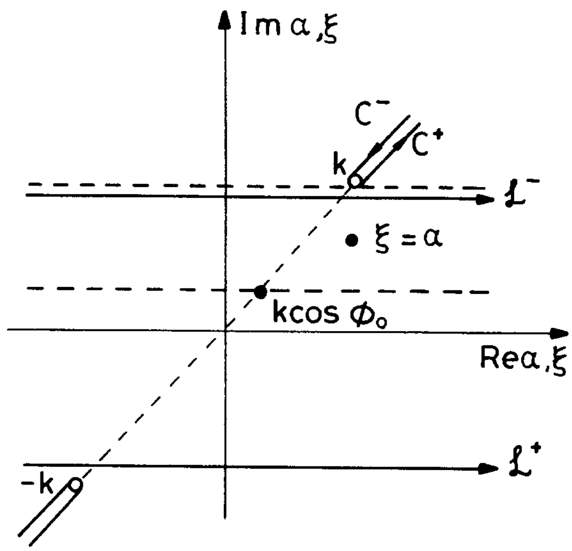

Fig. 2. Branch-cuts and integration lines in the complex plane.

is incident upon a strip $S = \{x \in (0, l)\,,\ y = 0\,,\ z \in (-\infty, \infty)\}$ which is soft at the top and hard at the bottom. In (2), k is the free space wave number which is assumed to have a small positive imaginary part, and $\phi_0 \in (0, \pi)$ is the angle of incidence.

For the scattered field $u(x, y)$ which satisfies the Helmholtz equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right) u(x, y) = 0 \quad , \quad y \neq 0, \tag{3}$$

it is appropriate to consider the following integral representation

$$u(x, y) = \begin{cases} \int\limits_{\mathcal{L}} A(\alpha) e^{iK(\alpha)y - i\alpha x} \mathrm{d}\alpha & , \quad y > 0 \\ \int\limits_{\mathcal{L}} B(\alpha) e^{-iK(\alpha)y - i\alpha x} \mathrm{d}\alpha & , \quad y < 0. \end{cases} \tag{4.a}$$

In (4.a), $\mathcal{L}$ is a straight line parallel to the real axis lying in the strip $\operatorname{Im}(k \cos \phi_0) < \operatorname{Im} \alpha < \operatorname{Im} k$, while $K(\alpha)$ stands for

$$K(\alpha) = \sqrt{k^2 - \alpha^2}. \tag{4.b}$$

The square-root function in (4.b) is defined in the complex α-plane cut as shown in Fig. 2, such that $K(0) = k$. The unknown spectral functions $A(\alpha)$ and $B(\alpha)$ appearing in (4.a) will be determined with the aid of the following boundary conditions and continuity relations:

$$u(x, +0) = -\exp(-ikx \cos \phi_0) \quad ; \quad x \in (0, l) \tag{5.a}$$

$$\frac{\partial}{\partial y} u(x, -0) = ik \sin \phi_0 \exp(-ikx \cos \phi_0) \quad ; \quad x \in (0, l) \tag{5.b}$$

$$u(x, +0) - u(x, -0) = 0 \quad ; \quad x \in \{(-\infty, 0) \cup (l, \infty)\} \tag{5.c}$$

$$\frac{\partial}{\partial y}u(x,+0) - \frac{\partial}{\partial y}u(x,-0) = 0 \quad ; \quad x \in \{(-\infty,0) \cup (l,\infty)\} \tag{5.d}$$

In order to obtain a unique solution it is also necessary to take into account the edge conditions at $x = 0$ and $x = l$. From Rawlins[11]

$$u(x,0) = \begin{cases} -1 + O\left(x^{1/4}\right) & x \to -0 \\ -\exp(-ikl\cos\phi_0) + O\left((x-l)^{1/4}\right) & x \to l+0 \end{cases} \tag{6.a}$$

$$\frac{\partial}{\partial y}u(x,0) = \begin{cases} O\left(x^{-3/4}\right) & x \to -0 \\ O\left((x-l)^{-3/4}\right) & x \to l+0 \end{cases} \tag{6.b}$$

Incorporating first (4.a) into (5.a–d) and then inverting the resulting integral equations one obtains

$$A(\alpha) = \Phi_1^-(\alpha) + e^{i\alpha l}\Phi_1^+(\alpha) - \frac{1}{2\pi i}\frac{\{\exp\left[il(\alpha - k\cos\phi_0)\right] - 1\}}{\alpha - k\cos\phi_0} \tag{7.a}$$

$$K(\alpha)B(\alpha) = \Phi_2^-(\alpha) + e^{i\alpha l}\Phi_2^+(\alpha) - \frac{k\sin\phi_0}{2\pi i}\frac{\{\exp\left[il(\alpha - k\cos\phi_0)\right] - 1\}}{\alpha - k\cos\phi_0} \tag{7.b}$$

$$A(\alpha) - B(\alpha) = 2P_1(\alpha) \tag{7.c}$$

$$A(\alpha) + B(\alpha) = 2P_2(\alpha)/K(\alpha) \tag{7.d}$$

In the above expressions $\Phi_{1,2}^{\pm}(\alpha)$ and $P_{1,2}(\alpha)$ stand for

$$\Phi_1^-(\alpha) = \frac{1}{2\pi}\int_{-\infty}^{0} u(x,+0)e^{i\alpha x}\mathrm{d}x \quad ; \quad \Phi_1^+(\alpha) = \frac{1}{2\pi}\int_{l}^{\infty} u(x,+0)e^{i\alpha(x-l)}\mathrm{d}x \tag{8.a,b}$$

$$\Phi_2^-(\alpha) = -\frac{1}{2\pi i}\int_{-\infty}^{0}\frac{\partial}{\partial y}u(x,-0)e^{i\alpha x}\mathrm{d}x \quad ;$$

$$\Phi_2^+(\alpha) = -\frac{1}{2\pi i}\int_{l}^{\infty}\frac{\partial}{\partial y}u(x,-0)e^{i\alpha(x-l)}\mathrm{d}x \tag{8.c,d}$$

$$P_1(\alpha) = \frac{1}{4\pi}\int_{0}^{l}\left[u(x,+0) - u(x,-0)\right]e^{i\alpha x}\mathrm{d}x \tag{8.e}$$

$$P_2(\alpha) = \frac{1}{4\pi i}\int_{0}^{l}\left[\frac{\partial}{\partial y}u(x,+0) - \frac{\partial}{\partial y}u(x,-0)\right]e^{i\alpha x}\mathrm{d}x \tag{8.f}$$

Owing to the analytical properties of Fourier integrals, the functions $\Phi_{1,2}^-(\alpha)$ and $\Phi_{1,2}^+(\alpha)$ are regular in the half-planes $\mathrm{Im}\,\alpha < \mathrm{Im}\,k$ and $\mathrm{Im}\,\alpha > \mathrm{Im}(k\cos\phi_0)$, respectively, while $P_{1,2}(\alpha)$ are entire functions. By using the edge conditions in (6.a,b) one can show easily that one has

$$P_1(\alpha) = O\left(\alpha^{-5/4}\right) \quad , \quad P_2(\alpha) = O\left(\alpha^{-1/4}\right), \quad |\alpha| \to \infty \ , \ \mathrm{Im}\,\alpha > \mathrm{Im}(k\cos\phi_0); \tag{9.a}$$

$$P_1(\alpha) = e^{i\alpha l} O\left(\alpha^{-5/4}\right) \quad , \quad P_2(\alpha) = e^{i\alpha l} O\left(\alpha^{-1/4}\right), \quad |\alpha| \to \infty \, , \, \mathrm{Im}\, \alpha < \mathrm{Im}\, k; \tag{9.b}$$

$$\Phi_1^-(\alpha) = -\frac{1}{2\pi i\alpha} + O\left(\alpha^{-5/4}\right) \quad , \quad \Phi_2^-(\alpha) = O\left(\alpha^{-1/4}\right), \quad |\alpha| \to \infty \, , \, \mathrm{Im}\, \alpha < \mathrm{Im}\, k; \tag{9.c}$$

$$\Phi_1^+(\alpha) = \frac{e^{-ikl\cos\phi_0}}{2\pi i\alpha} + O\left(\alpha^{-5/4}\right) \, , \, \Phi_2^+(\alpha) = O\left(\alpha^{-1/4}\right), \quad |\alpha| \to \infty \, , \, \mathrm{Im}\, \alpha > \mathrm{Im}(k\cos\phi_0). \tag{9.d}$$

The elimination of $A(\alpha)$ and $B(\alpha)$ among (7.a–d) yields the following MMWHE which is valid in the strip $\mathrm{Im}(k\cos\phi_0) < \mathrm{Im}\,\alpha < \mathrm{Im}\,k$:

$$\mathbf{G}(\alpha)\mathbf{P}(\alpha) = \mathbf{\Phi}^-(\alpha) + e^{i\alpha l}\mathbf{\Phi}^+(\alpha) + \mathbf{F}\frac{\{\exp\left[il(\alpha - k\cos\phi_0)\right] - 1\}}{\alpha - k\cos\phi_0}, \tag{10.a}$$

where

$$\mathbf{P}(\alpha) = \begin{bmatrix} P_1(\alpha) \\ P_2(\alpha) \end{bmatrix} \quad , \quad \mathbf{\Phi}^\pm(\alpha) = \begin{bmatrix} \Phi_1^\pm(\alpha) \\ \Phi_2^\pm(\alpha) \end{bmatrix} \tag{10.b}$$

with

$$\mathbf{G}(\alpha) = \begin{bmatrix} 1 & 1/K(\alpha) \\ -K(\alpha) & 1 \end{bmatrix} \tag{10.c}$$

and

$$\mathbf{F} = -\frac{1}{2\pi i}\begin{bmatrix} 1 \\ k\sin\phi_0 \end{bmatrix}. \tag{10.d}$$

3. Solution of the MMWHE

In order to obtain an explicit solution to (10.a), we have first of all to factorize the kernel matrix $\mathbf{G}(\alpha)$ as the product of two non-singular matrices, say $\mathbf{G}^+(\alpha)$ and $\mathbf{G}^-(\alpha)$, whose entries are regular functions of α in the upper and lower half-planes, respectively. The matrix $\mathbf{G}(\alpha)$ is of a special form which can be factorized via the Daniele-Khrapkov method. Indeed we have [7, 8, 12, 13]

$$\mathbf{G}^+(\alpha) = 2^{1/4}\begin{bmatrix} \cosh\chi(\alpha) & \sinh\chi(\alpha)/\gamma(\alpha) \\ \gamma(\alpha)\sinh\chi(\alpha) & \cosh\chi(\alpha) \end{bmatrix} \tag{11.a}$$

and

$$\mathbf{G}^-(\alpha) = \mathbf{G}^+(-\alpha), \tag{11.b}$$

where $\gamma(\alpha)$ and $\chi(\alpha)$ are given by

$$\gamma(\alpha) = \sqrt{\alpha^2 - k^2} = -iK(\alpha) \tag{11.c}$$

and

$$\chi(\alpha) = -\frac{i}{4}\arccos\frac{\alpha}{k} \quad , \quad \chi(-\alpha) = -\frac{i}{4}\left[\pi - \arccos\frac{\alpha}{k}\right]. \tag{11.d}$$

The function $\arccos \alpha/k$ is defined in the α-plane cut as shown in Fig. 2 ,with the condition $\arccos 0 = \pi/2$.

By considering the known asymptotics we can easily show that one has

$$\mathbf{G}^{+}(\alpha) \sim (4k)^{-1/4} \begin{bmatrix} \alpha^{1/4} & \alpha^{-3/4} \\ \alpha^{5/4} & \alpha^{1/4} \end{bmatrix} \tag{12.a}$$

and

$$\mathbf{G}^{-}(\alpha) \sim (4k)^{-1/4} \begin{bmatrix} (-\alpha)^{1/4} & (-\alpha)^{-3/4} \\ (-\alpha)^{5/4} & (-\alpha)^{1/4} \end{bmatrix}, \tag{12.b}$$

for $|\alpha| \to \infty$ in the upper and lower half-planes,respectively.

Now by using the method described in [14], the MMWHE in (10.a) can be reduced to the following pair of coupled integral equations:

$$\left[\mathbf{G}^{-}(\alpha)\right]^{-1} \mathbf{L}(\alpha) = \frac{1}{2\pi i} \int_{\mathcal{L}^{-}} e^{i\xi l} \left[\mathbf{G}^{-}(\xi)\right]^{-1} \mathbf{U}(\xi) \frac{d\xi}{\xi - \alpha}$$
$$- \frac{\left[\mathbf{G}^{-}(k\cos\phi_0)\right]^{-1} \mathbf{F}}{\alpha - k\cos\phi_0} + \mathbf{R} \tag{13.a}$$

$$\left[\mathbf{G}^{+}(\alpha)\right]^{-1} \mathbf{U}(\alpha) = -\frac{1}{2\pi i} \int_{\mathcal{L}^{+}} e^{-i\xi l} \left[\mathbf{G}^{+}(\xi)\right]^{-1} \mathbf{L}(\xi) \frac{d\xi}{\xi - \alpha}$$
$$+ \frac{\left[\mathbf{G}^{+}(k\cos\phi_0)\right]^{-1} \mathbf{F}}{\alpha - k\cos\phi_0} e^{-ikl\cos\phi_0} + \mathbf{S} \tag{13.b}$$

with

$$\mathbf{R} = r \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad , \quad \mathbf{S} = s \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{14.a,b}$$

and

$$\mathbf{L}(\alpha) = \mathbf{\Phi}^{-}(\alpha) - \frac{\mathbf{F}}{\alpha - k\cos\phi_0} \tag{14.c}$$

$$\mathbf{U}(\alpha) = \mathbf{\Phi}^{+}(\alpha) + \frac{\mathbf{F} e^{-ikl\cos\phi_0}}{\alpha - k\cos\phi_0}. \tag{14.d}$$

On using (9.c,d) one finds

$$L_1(\alpha) = O\left(\alpha^{-5/4}\right) \quad , \quad L_2(\alpha) = O\left(\alpha^{-1/4}\right), \quad |\alpha| \to \infty \, , \ \mathrm{Im}\,\alpha < \mathrm{Im}\,k; \tag{14.e}$$

$$U_1(\alpha) = O\left(\alpha^{-5/4}\right) \quad , \quad U_2(\alpha) = O\left(\alpha^{-1/4}\right), \quad |\alpha| \to \infty \, , \ \mathrm{Im}\,\alpha > \mathrm{Im}(k\cos\phi_0); \tag{14.f}$$

R and **S** appearing in (13.a) and (13.b) are unknown constant vectors resulting from the application of Liouville's theorem in the Wiener-Hopf procedure. Note that $\mathbf{U}(\alpha)$ is regular in the upper half-plane $\mathrm{Im}\,\alpha > \mathrm{Im}(k\cos\phi_0)$, while $\mathbf{L}(\alpha)$ is regular in the lower half-plane $\mathrm{Im}\,\alpha < \mathrm{Im}\,k$ except for a simple pole at $\alpha = k\cos\phi_0$. The positions of the integration lines $\mathcal{L}^{\pm}$ are indicated in Fig. 2.

The coupled system of Fredholm-type integral equations (13.a,b) is susceptible to a treatment by iterations. When l is large, the free terms in the right-hand side of (13.a) and (13.b) give the first-order solutions. Second-order solutions can then be obtained by replacing the unknown functions appearing in the integrands by their first -order approximations. Thus one can write

$$\mathbf{L}(\alpha) = \mathbf{L}^{(1)}(\alpha) + \mathbf{L}^{(2)}(\alpha) + \dots \tag{15.a}$$

$$\mathbf{U}(\alpha) = \mathbf{U}^{(1)}(\alpha) + \mathbf{U}^{(2)}(\alpha) + \dots \tag{15.b}$$

By assuming that the constants $\mathbf{R}$ and $\mathbf{S}$ can be written as

$$\mathbf{R} = \mathbf{R}^{(1)} + \mathbf{R}^{(2)} + \dots \tag{16.a}$$

$$\mathbf{S} = \mathbf{S}^{(1)} + \mathbf{S}^{(2)} + \dots \tag{16.b}$$

with

$$\mathbf{R}^{(j)} = r^{(j)} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad , \quad \mathbf{S}^{(j)} = s^{(j)} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{16.c,d}$$

the solution of the MMWHE in (10.a) reads

$$\mathbf{L}^{(j)}(\alpha) = [\mathbf{G}^{-}(\alpha)] \left[\mathbf{I}^{(j)}(\alpha) + \mathbf{R}^{(j)}\right] \quad , (j = 1,2) \tag{16.e}$$

$$\mathbf{U}^{(j)}(\alpha) = [\mathbf{G}^{+}(\alpha)] \left[\mathbf{J}^{(j)}(\alpha) + \mathbf{S}^{(j)}\right] \quad , (j = 1,2) \tag{16.f}$$

In (16.e,f) ,$\mathbf{I}^{(j)}(\alpha)$ and $\mathbf{J}^{(j)}(\alpha)$ stand for

$$\mathbf{I}^{(1)}(\alpha) = -\frac{[\mathbf{G}^{-}(k\cos\phi_0)]^{-1}}{\alpha - k\cos\phi_0}\mathbf{F} \tag{17.a}$$

$$\mathbf{J}^{(1)}(\alpha) = \frac{[\mathbf{G}^{+}(k\cos\phi_0)]^{-1}}{\alpha - k\cos\phi_0}\mathbf{F}e^{-ikl\cos\phi_0} \tag{17.b}$$

and

$$\mathbf{I}^{(2)}(\alpha) = \frac{1}{2\pi i}\int\limits_{\mathcal{L}^-} e^{i\xi l}\frac{[\mathbf{G}^{-}(\xi)]^{-1}}{\xi - \alpha}\mathbf{U}^{(1)}(\xi)\mathrm{d}\xi \tag{18.a}$$

$$\mathbf{J}^{(2)}(\alpha) = -\frac{1}{2\pi i}\int\limits_{\mathcal{L}+} e^{-i\xi l}\frac{[\mathbf{G}^{+}(\xi)]^{-1}}{\xi - \alpha}\mathbf{L}^{(1)}(\xi)\mathrm{d}\xi. \tag{18.b}$$

Now consider the integral in (18.a) and rearrange it as follows:

$$\mathbf{I}^{(2)}(\alpha) = \frac{1}{4\pi i}\int\limits_{\mathcal{L}^-} e^{i\xi l}\mathbf{G}^{+}(\xi)\begin{bmatrix} 1 & -1/i\gamma(\xi) \\ i\gamma(\xi) & 1 \end{bmatrix}\frac{\mathbf{U}^{(1)}(\xi)}{\xi - \alpha}\mathrm{d}\xi. \tag{19}$$

Since $l > 0$, according to Jordan's lemma, the integration line $\mathcal{L}^-$ can be deformed into the branch-cut $\mathcal{C}^+ + \mathcal{C}^-$ lying in the upper half-plane and $\mathbf{I}^{(2)}(\alpha)$ can now be expressed as

$$\mathbf{I}^{(2)}(\alpha) = \frac{1}{\alpha - k\cos\phi_0}\int\limits_{\mathcal{C}+} e^{i\xi l}\mathbf{M}(\xi)\begin{bmatrix} (\xi - k)^{1/2} \\ (\xi - k)^{-1/2} \end{bmatrix}\left(\frac{1}{\xi - \alpha} - \frac{1}{\xi - k\cos\phi_0}\right)\mathrm{d}\xi, \tag{20.a}$$

where $\mathbf{M}(\alpha)$ is a matrix-valued function regular in the upper half-plane, and given by

$$\mathbf{M}(\alpha) = \frac{1}{2\pi}\mathbf{G}^{+}(\alpha)\begin{bmatrix} 0 & \tilde{U}_2^{(1)}(\alpha)/\sqrt{\alpha+k} \\ \tilde{U}_1^{(1)}(\alpha)\sqrt{\alpha+k} & 0 \end{bmatrix} \tag{20.b}$$

with

$$\tilde{U}_{1,2}^{(1)}(\alpha) = U_{1,2}^{(1)}(\alpha)(\alpha - k\cos\phi_0). \tag{20.c}$$

The integral in (20.a) can finally be reduced to the following one along the positive real axis:

$$\mathbf{I}^{(2)}(\alpha) = \frac{e^{ikl}}{\alpha - k\cos\phi_0}\int_0^\infty e^{itl}\mathbf{M}(t+k)\begin{bmatrix} t^{1/2} \\ t^{-1/2} \end{bmatrix}\left[\frac{1}{t+(k-\alpha)} - \frac{1}{t+k(1-\cos\phi_0)}\right]\mathrm{d}t. \tag{21}$$

When the acoustical width kl of the soft/hard strip is large ($kl >> 1$), the main contribution to the integral in (21) comes from the end point $t = 0$. Hence $\mathbf{M}(t+k)$ can be taken out from the integral by assigning its value at $t = 0$. The resulting integral can be expressed in terms of the modified Fresnel integral

$$F(z) = -2i\sqrt{z}e^{-iz}\int_{\sqrt{z}}^\infty e^{it^2}\mathrm{d}t \tag{22}$$

to give

$$\mathbf{I}^{(2)}(\alpha) \sim \sqrt{k\pi}\frac{e^{ikl}}{\sqrt{kl}}\frac{e^{i\pi/4}}{(\alpha - k\cos\phi_0)}\mathbf{M}(k)\begin{bmatrix} F(kl(1-\cos\phi_0)) - F(kl(1-\alpha/k)) \\ \dfrac{F(kl(1-\alpha/k))}{k-\alpha} - \dfrac{F(kl(1-\cos\phi_0))}{k(1-\cos\phi_0)} \end{bmatrix}. \tag{23}$$

By proceeding similarly, one obtains for $\mathbf{J}^{(2)}(\alpha)$ the following result:

$$\mathbf{J}^{(2)}(\alpha) \sim \sqrt{k\pi}\frac{e^{ikl}}{\sqrt{kl}}\frac{e^{i\pi/4}}{(\alpha - k\cos\phi_0)}\mathbf{N}(-k)\begin{bmatrix} F(kl(1+\cos\phi_0)) - F(kl(1+\alpha/k)) \\ \dfrac{F(kl(1+\alpha/k))}{k+\alpha} - \dfrac{F(kl(1+\cos\phi_0))}{k(1+\cos\phi_0)} \end{bmatrix}, \tag{24.a}$$

where $\mathbf{N}(\alpha)$ stands for

$$\mathbf{N}(\alpha) = \frac{1}{2\pi}\mathbf{G}^{-}(\alpha)\begin{bmatrix} 0 & \tilde{L}_2^{(1)}(\alpha)/\sqrt{k-\alpha} \\ \tilde{L}_1^{(1)}(\alpha)\sqrt{k-\alpha} & 0 \end{bmatrix} \tag{24.b}$$

with

$$\tilde{L}_{1,2}^{(1)}(\alpha) = L_{1,2}^{(1)}(\alpha)(\alpha - k\cos\phi_0). \tag{24.c}$$

4. Determination of the Constants and Diffracted Fields

The spectral coefficients appearing in (4.a) can be expressed in terms of $\mathbf{L}^{(\mathbf{j})}(\alpha)$ and $\mathbf{U}^{(\mathbf{j})}(\alpha)$ as follows:

$$\begin{bmatrix} A(\alpha) \\ B(\alpha) \end{bmatrix} = \sum_{j=1}^{2} \begin{bmatrix} A^{(j)}(\alpha) \\ B^{(j)}(\alpha) \end{bmatrix} + \dots$$

$$= \sum_{j=1}^{2} \begin{bmatrix} 1 & 0 \\ 0 & 1/i\gamma(\alpha) \end{bmatrix} \left\{ \mathbf{L}^{(\mathbf{j})}(\alpha) + \mathbf{e}^{\mathbf{i}\alpha\mathbf{l}} \mathbf{U}^{(\mathbf{j})}(\alpha) \right\} + \dots \tag{25}$$

Here, the spectral coefficients for $j = 1$ and $j = 2$ correspond to the singly and doubly diffracted fields, respectively. Note that the first and second terms in the braces are related to the diffracted fields emanating from the edges O and Q, respectively.

The unknown constant vectors $\mathbf{R}^{(\mathbf{j})}$ and $\mathbf{S}^{(\mathbf{j})}$ will now be specified by taking into account the edge conditions. From (7.a,b) and (14.c,d,e,f) it is known that the spectral quantities $A(\alpha)$ and $B(\alpha)$ should behave like $O\left(\alpha^{-5/4}\right)$ as $|\alpha| \to \infty$ in the strip $\text{Im}(k\cos\phi_0) < \text{Im}\,\alpha < \text{Im}\,k$. Letting $|\alpha| \to \infty$ in (25) gives:

$$\begin{bmatrix} A^{(j)}(\alpha) \\ B^{(j)}(\alpha) \end{bmatrix} = (4k)^{-1/4} \left\{ \begin{bmatrix} -\tilde{I}_1^{(j)} + r^{(j)} \\ \pm i(-\tilde{I}_1^{(j)} + r^{(j)}) \end{bmatrix} (-\alpha)^{-3/4} + e^{i\alpha l} \begin{bmatrix} \tilde{J}_1^{(j)} + s^{(j)} \\ \mp i(\tilde{J}_1^{(j)} + s^{(j)}) \end{bmatrix} \alpha^{-3/4} \right\}$$
$$+ O\left(\alpha^{-5/4}\right) \tag{26.a}$$

with the upper(lower) sign holding as $\text{Re}\,\alpha \to \infty (\text{Re}\,\alpha \to -\infty)$ in the strip $\text{Im}(k\cos\phi_0) <$ $\text{Im}\,\alpha < \text{Im}\,k$. $\tilde{I}_1^{(j)}$ and $\tilde{J}_1^{(j)}$ appearing in (26.a) stand for

$$\tilde{I}_1^{(j)} = \lim_{\alpha\to\infty} \alpha I_1^{(j)}(\alpha) \quad , \quad \tilde{J}_1^{(j)} = \lim_{\alpha\to\infty} \alpha J_1^{(j)}(\alpha). \tag{26.b}$$

The admissible edge field behaviour is guaranteed by choosing

$$r^{(j)} = \tilde{I}_1^{(j)} \quad , \quad s^{(j)} = -\tilde{J}_1^{(j)}. \tag{27}$$

A) SINGLY DIFFRACTED FIELDS

Consider first the singly diffracted field by the edge O, namely

$$u_O = \int_{\mathcal{L}} L_1^{(1)}(\alpha) \exp\left[iK(\alpha)y - i\alpha x\right] \mathrm{d}\alpha \ ; \qquad y > 0 \tag{28}$$

If k is real, the integration line lies along the real axis with indentations above $\alpha = -k$ and $\alpha = k\cos\phi_0$ and below $\alpha = k$. The substitutions $\alpha = -k\cos\theta$, $x = \rho\cos\phi$, $y = \rho\sin\phi$ with $0 < \phi < \pi$, enable one to write

$$u_O = \int_{\Gamma} L_1^{(1)}(-k\cos\theta) \exp\left[ik\rho\cos(\theta - \phi)\right] k\sin\theta \mathrm{d}\theta. \tag{29}$$

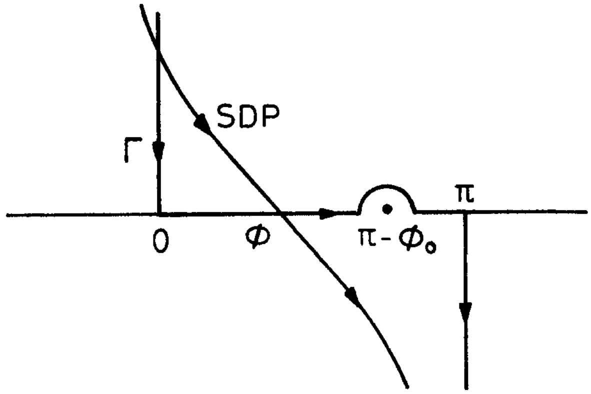

Fig. 3. The integration line Γ and the steepest descent path in the complex θ-plane.

where Γ is the new integration path shown in Fig. 3. During the deformation of Γ into the steepest descent path (SDP) through the saddle point $\theta = \phi$, the pole at $\theta = \pi - \phi_0$ is intercepted if $\phi < \pi - \phi_0$. The corresponding residue contribution is:

$$-\exp\left[ik\rho\cos(\pi - \phi_0 - \phi)\right] H(\pi - \phi_0 - \phi) = u^r H(\pi - \phi_0 - \phi) \tag{30.a}$$

where u^r is the geometrical optics reflected field and H denotes the unit step function. In the case $y < 0$, or $\pi < \phi < 2\pi$, it is found in the same manner that the residue contribution is given by

$$-\exp\left[ik\rho\cos(\pi - \phi_0 + \phi)\right] H(\phi - \pi - \phi_0) = -u^i H(\phi - \pi - \phi_0). \tag{30.b}$$

Returning to the case $y > 0$, consider now the integral along the steepest descent path, viz.

$$u_O^d = \int_{SDP} L_1^{(1)}(-k\cos\theta)\exp\left[ik\rho\cos(\theta - \phi)\right] k\sin\theta \mathrm{d}\theta. \tag{31}$$

Where the superscript d refers to the diffracted field. The integrand in (31) has a simple pole at $\theta = \pi - \phi_0$ and after a tedious calculation the integral is written as

$$u_O^d \sim -\frac{2^{1/2}}{8\pi i}\int_{SDP} W(\phi_0,\theta)\left[\sec\frac{1}{2}(\theta + \phi_0) + \sec\frac{1}{2}(\theta - \phi_0)\right]\exp\left[ik\rho\cos(\theta - \phi)\right]\mathrm{d}\theta \tag{32}$$

where $W(\phi_0, \theta)$ is given by

$$\begin{aligned} W(\phi_0,\theta) &= \frac{1}{\cos\frac{1}{2}\phi_0\cos\frac{1}{2}\theta}\left\{\sin\frac{1}{4}\phi_0\cos\frac{1}{4}\theta\sin\theta + \sin\phi_0\cos\frac{1}{4}\phi_0\sin\frac{1}{4}\theta\right. \\ &\quad \left. - \sin\frac{1}{4}\phi_0\sin\frac{1}{4}\theta(\cos\phi_0 + \cos\theta)\right\} \\ &= 2\frac{\sin\frac{1}{4}\phi_0\sin\frac{1}{4}\theta}{\cos\frac{1}{2}\phi_0\cos\frac{1}{2}\theta}\left[1 + \cos\frac{1}{2}\phi_0 + \cos\frac{1}{2}\theta\right]. \end{aligned} \tag{33}$$

As $k\rho \to \infty$, the main contribution to the integral (32) comes from a vicinity of the saddle-point $\theta = \phi$. Thus, one may replace $W(\phi_0, \theta)$ by $W(\phi_0, \phi)$ and take $W(\phi_0, \phi)$ outside of the integral (32). The resulting integral can be expressed in terms of the modified Fresnel integral to give

$$u_O^d(\rho,\phi) \sim u^i(O)\frac{e^{i\pi/4}}{4\sqrt{\pi}}\frac{e^{ik\rho}}{\sqrt{k\rho}}\left[\sec\frac{1}{2}(\phi+\phi_0)F\left(2k\rho\cos^2\frac{1}{2}(\phi+\phi_0)\right)\right.$$
$$\left.+\sec\frac{1}{2}(\phi-\phi_0)F\left(2k\rho\cos^2\frac{1}{2}(\phi-\phi_0)\right)\right]W(\phi_0,\phi). \tag{34}$$

By repeating a similar analysis for the case $y < 0$, it is found that (34) remains valid for $\pi < \phi < 2\pi$.

Away from the shadow boundaries $\phi = \pi \mp \phi_0$, one may set $F = 1$ in (34) and this expression reduces to

$$u_O^d(\rho,\phi) \sim u^i(O)D(\phi_0,\phi)\frac{e^{ik\rho}}{\sqrt{k\rho}} \tag{35.a}$$

where $D(\phi_0,\phi)$ is the diffraction coefficient related to a soft/hard half plane given by

$$D(\phi_0,\phi) = 2\frac{e^{i\pi/4}}{\sqrt{\pi}}\frac{\sin\frac{1}{4}\phi_0\sin\frac{1}{4}\phi}{\cos\phi_0+\cos\phi}\left[1+\cos\frac{1}{2}\phi_0+\cos\frac{1}{2}\phi\right]. \tag{35.b}$$

This result is identical to that given by Senior[15] who considered the soft/hard half plane problem through the Maliuzhinetz method.

Notice that the sum of the residue contributions (30.a) and (30.b)

$$u^r H(\pi-\phi_0-\phi) - u^i H(\phi-\pi-\phi_0) = u^{go} - u^i \tag{36}$$

is recognized as the geometrical optics field u^{go} minus the incident field u^i. Thus, the field u_O^d of (32) is to be considered as the diffracted field due to the diffraction by the soft/hard half plane $x > 0$, $y = 0$.

Finally, it is worth to point out that the terms $\sec\frac{1}{2}(\phi\pm\phi_0)F\left(2k\rho\cos^2\frac{1}{2}(\phi\pm\phi_0)\right)$ in (34) are discontinuous across the shadow boundaries $\phi = \pi \mp \phi_0$. It can be readily seen that the corresponding discontinuities of the diffracted field u_O^d are precisely compensated by the discontinuities of the geometrical optics field.

A similar analysis shows that the singly diffracted field by the edge Q, say u_Q^d, can be obtained from (34) by making the following substitutions

$$u^i(O) \to u^i(Q)\ ,\ \rho \to r\ ,\ \phi_0 \to \pi-\phi_0\ ,\ \phi \to \pi-\psi\,;$$

where r and $\psi \in (-\pi,\pi)$ are the polar coordinates measured from Q.

B) DOUBLY DIFFRACTED FIELDS

The doubly diffracted field by the edge O, say u_{QO}, will be obtained by asymptotically evaluating the following integral

$$u_{QO} = \int_{\mathcal{L}} L_1^{(2)}(\alpha)\exp\left[iK(\alpha)y - i\alpha x\right]d\alpha\ ; \qquad y > 0. \tag{37}$$

Through the substitutions $\alpha = -k\cos\theta$, $x = \rho\cos\phi$, $y = \rho\sin\phi$ with $0 < \phi < \pi$, the integral (37) passes into

$$u_{QO} = \int_{\Gamma} L_1^{(2)}(-k\cos\theta)\exp\left[ik\rho\cos(\theta-\phi)\right] k\sin\theta \mathrm{d}\theta \tag{38}$$

where Γ is shown in Fig. 3. Next, Γ is deformed into the steepest descent path (SDP) through the saddle point $\theta = \phi$. No poles are intercepted in the deformation, since $L_1^{(2)}(\alpha)$ is analytic in the half-plane $\mathrm{Im}\,\alpha < \mathrm{Im}\,k$. By use of the saddle point formula one obtains

$$\begin{aligned} u_{QO} &= \int_{SDP} L_1^{(2)}(-k\cos\theta)\exp\left[ik\rho\cos(\theta-\phi)\right] k\sin\theta \mathrm{d}\theta \\ &\sim \sqrt{2\pi}e^{-i\pi/4}L_1^{(2)}(-k\cos\phi)k\sin\phi\frac{e^{ik\rho}}{\sqrt{k\rho}} \end{aligned} \tag{39}$$

By further evaluation of $L_1^{(2)}(-k\cos\phi)$ one gets finally

$$\begin{aligned} u_{QO}(\rho,\phi) = &-\frac{2^{-1/2}}{(\cos\phi_0+\cos\phi)}\frac{e^{ikl}}{\sqrt{kl}}\frac{e^{ik\rho}}{\sqrt{k\rho}} \times \\ &\left\{\frac{\tilde{U}_1^{(1)}(k)}{2}\left[\left[\sin\phi\cos(\phi/4)+4\sin(\phi/4)\right]\left[F(kl(1-\cos\phi_0))-F(kl(1+\cos\phi))\right]\right.\right. \\ &\left.+\sin(\phi/4)(\cos\phi_0+\cos\phi)\left[1-F(kl(1-\cos\phi_0))\right]\right] \\ &+\frac{\tilde{U}_2^{(1)}(k)}{k}\left[\sin\phi\cos(\phi/4)\frac{F(kl(1+\cos\phi))}{1+\cos\phi}\right. \\ &\left.\left.+\left[(\cos\phi_0+\cos\phi)\sin(\phi/4)-\sin\phi\cos(\phi/4)\right]\frac{F(kl(1-\cos\phi_0))}{1-\cos\phi_0}\right]\right\} \end{aligned} \tag{40.a}$$

where $\tilde{U}_1^{(1)}(k)$ and $\tilde{U}_2^{(1)}(k)$ stand for

$$\tilde{U}_1^{(1)}(k) = -\frac{u^i(Q)}{\sqrt{2\pi i}}\left\{\frac{1}{4}\sin\phi_0\cos\left(\frac{\pi-\phi_0}{4}\right)+\left[1-\frac{1}{4}(1-\cos\phi_0)\right]\sin\left(\frac{\pi-\phi_0}{4}\right)\right\} \tag{40.b}$$

$$\tilde{U}_2^{(1)}(k) = -k\frac{u^i(Q)}{\sqrt{2\pi i}}\left\{\sin\phi_0\cos\left(\frac{\pi-\phi_0}{4}\right)-(1-\cos\phi_0)\sin\left(\frac{\pi-\phi_0}{4}\right)\right\} \tag{40.c}$$

For angles of incidence and observation away from grazing, the Fresnel integral appearing in (40.a) can be replaced by its asymptotic expression valid for large arguments, namely

$$F(z) \sim 1+\frac{1}{2iz} \quad , \quad z\to\infty \tag{41}$$

In such a case the doubly diffracted field by the edge O can be cast into the following form:

$$u_{QO}(\rho,\phi) \sim u^i(Q)\left[T_{es}^{(1)}(\phi_0)\frac{e^{ikl}}{\sqrt{kl}}T_{se}^{(1)}(\phi)+T_{es}^{(2)}(\phi_0)\frac{e^{ikl}}{(kl)^{3/2}}T_{se}^{(2)}(\phi)\right]\frac{e^{ik\rho}}{\sqrt{k\rho}} \tag{42}$$

with

$$T_{es}^{(1)}(\phi_0) = \frac{e^{i\pi/4}}{\sqrt{2\pi}} \frac{\sin\frac{1}{4}(\pi - \phi_0)}{\sin\frac{1}{2}\phi_0} \tag{43.a}$$

$$T_{se}^{(1)}(\phi) = \frac{e^{i\pi/4}}{\sqrt{2\pi}} \frac{\sin\frac{1}{4}\phi}{\cos\frac{1}{2}\phi} \tag{43.b}$$

$$T_{es}^{(2)}(\phi_0) = \frac{i}{2\sqrt{2\pi}} \frac{\sin\frac{1}{4}(\pi - \phi_0)\left(2 + \sin\frac{1}{2}\phi_0\right)}{1 - \cos\phi_0} \tag{43.c}$$

$$T_{se}^{(2)}(\phi) = \frac{i}{2\sqrt{2\pi}} \frac{\sin\frac{1}{4}\phi\left(2 + \cos\frac{1}{2}\phi\right)}{1 + \cos\phi}. \tag{43.d}$$

Note that $T_{es}^{(1)}(\phi_0)$ and $T_{se}^{(1)}(\phi)$ are related to the diffraction coefficient D defined in (35.b) via

$$T_{es}^{(1)}(\phi_0) = 2^{-1/2} D(\pi - \phi_0, 2\pi) \tag{44.a}$$

$$T_{se}^{(1)}(\phi) = 2^{-1/2} D(2\pi, \phi). \tag{44.b}$$

By use of these results, the first term in (42) can be rewritten as

$$\frac{1}{2} u^i(Q) D(\pi - \phi_0, 2\pi) \frac{e^{ikl}}{\sqrt{kl}} D(2\pi, \phi) \frac{e^{ik\rho}}{\sqrt{k\rho}}. \tag{45}$$

The interpretation of (45) is obvious: the incident field u^i is diffracted at Q along the bottom side of the strip, and subsequently diffracted at O at the angle ϕ. The factor $\frac{1}{2}$ in (45) stems from the fact that only the singly diffracted field u_Q along the bottom side contributes to the doubly diffracted field (45); along the soft upper side the field u_Q vanishes.

By introducing the slope diffraction coefficient defined by

$$D'(\phi) = \frac{\partial}{\partial \phi_0} D(\phi_0, \phi)|_{\phi_0 = 0} = \frac{e^{i\pi/4}}{2\sqrt{\pi}} \frac{\sin\frac{1}{4}\phi\left(2 + \cos\frac{1}{2}\phi\right)}{1 + \cos\phi}, \tag{46}$$

$T_{es}^{(2)}$ and $T_{se}^{(2)}$ can be expressed as

$$T_{es}^{(2)}(\phi_0) = 2^{-1/2} e^{i\pi/4} D'(\pi - \phi_0) \quad , \quad T_{se}^{(2)}(\phi) = 2^{-1/2} e^{i\pi/4} D'(\phi). \tag{47.a,b}$$

Then the second term in (42) can be cast into the following form

$$\frac{1}{2} i u^i(Q) D'(\pi - \phi_0) \frac{e^{ikl}}{(kl)^{3/2}} D'(\phi) \frac{e^{ik\rho}}{\sqrt{k\rho}}. \tag{48}$$

The latter result admits the following interpretation. The singly diffracted field u_Q along the upper side of the strip, can be shown to have a normal derivative at O, given by

$$\frac{\partial u_Q}{\partial y}(O+) = \frac{\partial u_Q}{\partial y}(l, \pi) \sim u^i(Q) D'(\pi - \phi_0) \frac{e^{ikl}}{k^{1/2} l^{3/2}}. \tag{49}$$

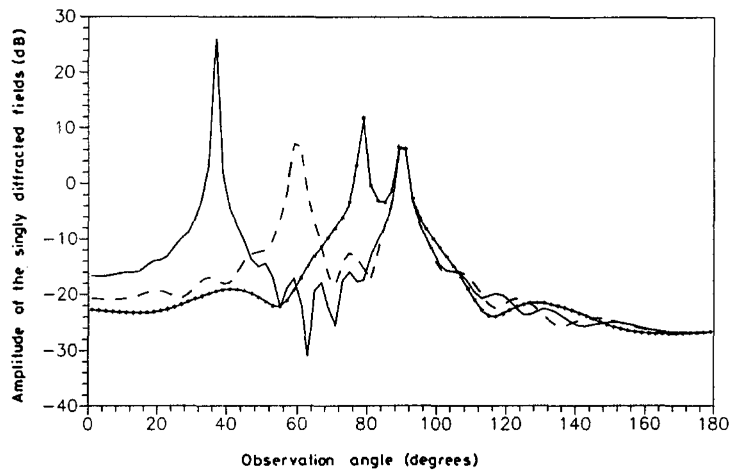

Fig. 4. $20 \log |u^{(1)}|$ versus the observation angle for a soft/hard strip. $\rho = 10\lambda$, $\phi_0 = 90°$, $* * * *$ $l = 2\lambda$, $- - - -$ $l = 5\lambda$, —— $l = 8\lambda$.

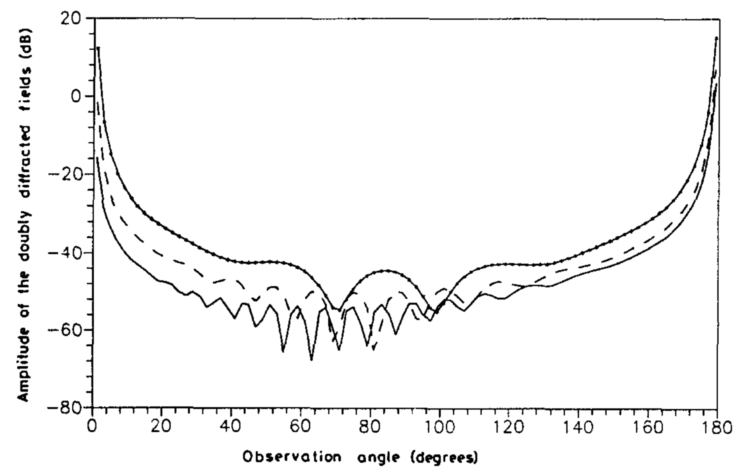

Fig. 5. $20 \log |u^{(2)}|$ versus the observation angle for a soft/hard strip. $\rho = 10\lambda$, $\phi_0 = 90°$, $* * * *$ $l = 2\lambda$, $- - - -$ $l = 5\lambda$, —— $l = 8\lambda$.

According to Keller[16], the doubly diffracted field due to the diffraction of u_Q at O is given by the normal derivative (49) multiplied by $(i/k)D'(\phi)e^{ik\rho}/\sqrt{k\rho}$. Thus the result (48) is precisely recovered, apart from the factor $\frac{1}{2}$ which stems from the fact that only the singly diffracted field u_Q along the upper side of the strip contributes to the doubly diffracted field; along the hard bottom side, the normal derivative $\partial u_Q/\partial y$ vanishes.

As for the singly diffracted field, the doubly diffracted wave by the edge Q can be obtained from the above result by making the following substitutions :

$$Q \leftrightarrow O \quad , \quad \phi_0 \to \pi - \phi_0 \quad , \quad \phi \to \pi - \psi \quad , \quad \rho \to r.$$

Figs. 4 and 5 show the variations of the singly and doubly diffracted fields, respectively, for different values of the strip width l. It is seen that the amplitude of the doubly diffracted field decreases with increasing strip width.

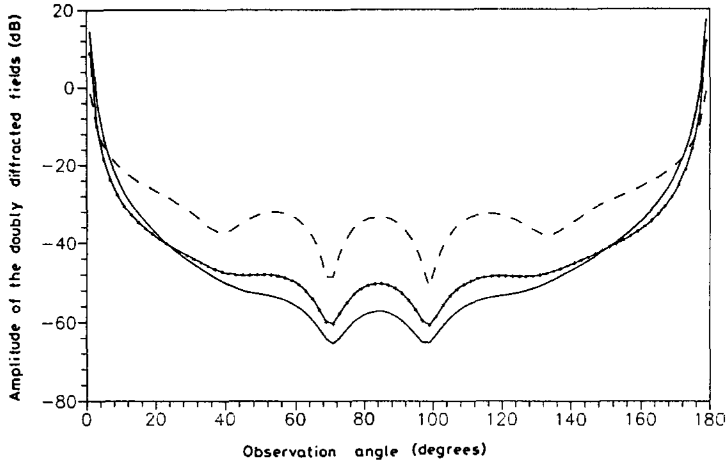

Fig. 6. $20\log|u^{(2)}|$ versus the observation angle for soft,hard and soft/hard strips. $\rho = 10\lambda$, $l = 2\lambda$; $\phi_0 = 90^\circ$; * * * * * soft/hard strip, —— soft strip; — — — hard strip.

Fig. 6 displays the amplitudes of the doubly diffracted fields for the completely soft, completely hard and soft/hard strips, respectively. The curves related to soft and hard strips were computed by using the results given in [17,Ch.4]. From this graph one can conclude that a strip with a soft boundary condition on one side and a rigid boundary condition on the other, gives better attenuation of the doubly diffracted fields than the completely rigid strip. This fact was also observed by Rawlins [11] for the singly diffracted fields.

5. Concluding Remarks

The boundary value problem related to the diffraction of plane waves by a strip with Dirichlet and Neumann boundary conditions on its upper and lower faces, respectively, is considered through the Wiener-Hopf approach. By performing the Wiener-Hopf factorization of the kernel matrix via the Daniele-Khrapkov method, the related MMWHE is first reduced to a pair of coupled Fredholm integral equations of the second kind and then solved approximately by iterations. The use of the Daniele-Khrapkov method yields a solution containing unknown constants which can be specified by taking into account the edge conditions. One should note that these constants do not appear when the factorisation of the kernel matrix is accomplished through the Wiener-Hopf-Hilbert method.

From computational results, it is observed that a soft/hard strip gives better attenuation of the diffracted field then the completely rigid strip.

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